

# **Turbulence Modeling in Stratified Flows Subject to Advective Buoyancy Fluxes**

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## **LONG-TERM GOALS**

The goal of this work is to examine in detail how turbulence evolves in the presence of both vertical stratification and horizontal (straining) buoyancy fluxes. Horizontal density gradients are a ubiquitous feature of estuarine and coastal flows, as are sheared velocity profiles. The combination of these two features produces a buoyancy flux which can be either stabilizing or destabilizing, depending on the orientation of the currents relative to the density gradient (see, e.g. Simpson et al. 1990). The research being pursued under this grant is seeking to develop robust parameterizations of turbulent mixing that can be used with circulation models and provide efficient and accurate solutions to the turbulent closure problem.

## **OBJECTIVES**

Specific objectives of the proposed work involve the continued analysis of the 1999 San Francisco Bay turbulence data set, and the comparison of these data with numerical predictions of mean velocity, density stratification and turbulence quantities. To examine the interaction between turbulence and stratification in more detail than is available through field inquire, a large eddy simulation of the estuary channel will be pursued. Using both the large eddy simulation results and the field observations, robust turbulence parameterizations will be developed, which will be evaluated using a Reynolds-averaged model of an estuarine water column. In the first year, the goal is to develop the modeling capabilities – both the water column and the large eddy simulations. In the second year, we will begin to pursue model-data comparisons.

## **APPROACH**

We begin with analysis of an existing data set (SC99), but will be developing both a Reynolds-averaged model and a large-eddy simulation of stratified channel flow that includes the influence of the horizontal buoyancy flux. The large eddy simulation will be used to look in detail at the structure of the turbulence and the competition between shear and buoyancy production. These results, in combination with the field data, will be used to develop improved turbulence parameterizations. The Reynolds-averaged model will be used to evaluate the performance of these closures, specifically through comparison of the predicted mean velocity and density fields with those measured in the field.

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## WORK COMPLETED

This project was a late start in FY03 (Start date: 3/3/2003). In the first few months of the project, we have completed analysis of the structure of the estuarine bottom boundary layer and the structure and variability of the turbulent kinetic energy (TKE) within the BBL. These results are described in a manuscript submitted to the *Journal of Physical Oceanography* (Stacey and Ralston, submitted). Currently, I have begun analysis of turbulent lengthscales, specifically examining whether tidal asymmetries develop due to the influence of straining. In parallel, I have begun to work with a Reynolds-averaged model of the water column. For now, I am examining simple, integrated models of TKE and BBL height using the scaling results described in the next section.

## RESULTS

As outlined in the manuscript submitted to *Journal of Physical Oceanography*, we have been working to establish scaling relationships that describe the variability of turbulence in the bottom boundary layer as it responds to tidal straining. We assume that the bottom boundary layer is well-mixed (or nearly so), and that the turbulent stress distribution can be described by two quantities: the boundary layer height ( $H_{BBL}$ ) and the friction velocity ( $u_*$ ), with linear variation of the stresses from  $u_*^2$  at the bed to 0 at  $H_{BBL}$ . To examine the influence of the straining of the density field on these quantities, we define the effective (horizontal) buoyancy flux as:

$$B_h = N_x^2 H_{BBL} u_* / \kappa$$

where  $N_x^2 = -(g / \rho_0) \partial \rho / \partial x$ ; this approach will be used for scaling purposes in the next sections.

### Ebb tide scaling

On ebb tides, shear in the water column is stabilizing due to the straining of the density field, but destabilizing through shear production. The top of the bottom boundary layer will be determined by the point at which the ‘available’ vertical buoyancy flux due to turbulent production is in balance with the stabilizing buoyancy flux from the straining of the density field. That is,

$$B_v = R_f P \approx B_h$$

Which can be solved for the boundary layer height to give:

$$H_{BBL} = \sqrt{R_f} u_* / N_x$$

The fact that shear production is the dominant source of turbulence suggests a drag-based formulation of the friction velocity:

$$u_* \approx C_d^{1/2} U$$

The comparison between this model and the observations are shown in figure 1; the agreement indicates that a scaling approach may be successful at predicting bottom boundary layer development.

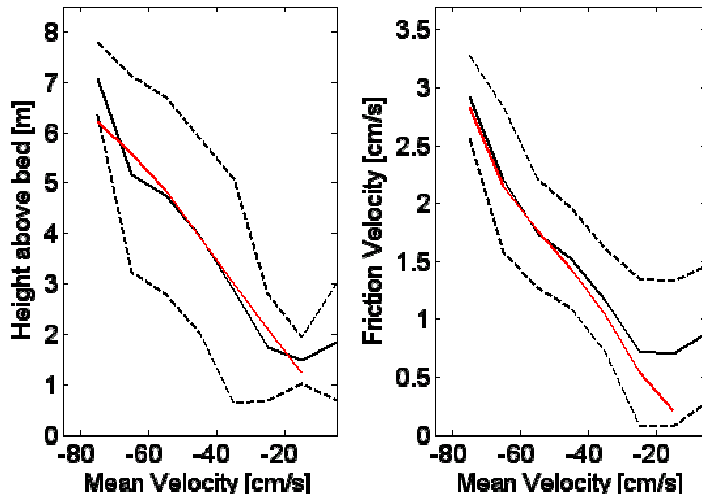
### Flood tide scaling

On flood tides, the shear in the water column is destabilizing both through the shear production and through the strain-induced buoyancy flux, except in regions where the shear reverses sign. In the Suisun Cutoff data set, this occurs mid-column due to upstream shoals providing a source of near-surface low-momentum fluid. Above this mid-column maximum, the stabilizing effects of straining exceed the local shear production, and the water column stratifies; below this point, both buoyancy

and shear production are sources of turbulent mixing. The top of the bottom boundary layer in this case, therefore, will be at the velocity maximum (for many cases this will be at the free surface). The friction velocity scaling depends on the dominant forcing mechanism, with the limiting cases being a drag-coefficient approach (shear-dominated production; defined above) and a free convection scaling (buoyancy-dominated production):

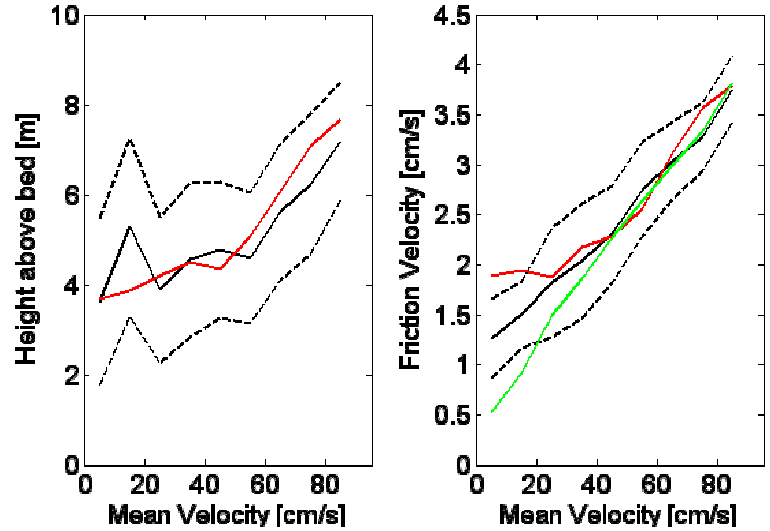
$$u_* \sim (U N_x^2 H_{BBL}^2)^{1/3}$$

The comparison between this scaling prediction and the observations are shown in figure 2. The boundary layer height is well-predicted by the height of the velocity maximum; the friction velocity, however, could be appropriately scaled by either a drag-coefficient or free convection approach over most the data set presented here. At lower mean velocities, it appears that the free convection scaling may be more appropriate, but more evaluation of this point is necessary.



*Figure 1: Comparison of ebb tide scalings for (a)  $H_{BBL}$ ; and (b)  $u_*$ . Solid black line is average of data from Suisun Cutoff (bin-averaged by depth-averaged velocity), dashed lines are plus and minus one standard deviation. Red line is scaling prediction with  $R_f=0.2$  and  $C_d=0.0018$ .*

*Figure 2: Comparison of flood tide scalings for (a)  $H_{BBL}$ ; and (b)  $u_*$ . Solid black line is average of data from Suisun Cutoff (bin-averaged by depth-averaged velocity), dashed lines are plus and minus one standard deviation. Red line is scaling prediction based on free convection; Green line is alternative friction velocity scaling based on drag coefficient,  $C_d=0.0022$ .*



In an unstratified bottom boundary layer, the strain-induced buoyancy flux is converted into a vertical buoyancy flux, which is a sink of turbulent kinetic energy on ebb tides and a source of turbulent kinetic energy on floods. The importance of this component of the turbulent kinetic energy budget relative to the shear production term is presented in figure 3. Here we see that on ebb tides, the strain-induced buoyancy flux is comparable to the shear production at the top of the boundary layer, which is expected based on the success of the scaling approach to the boundary layer height previously presented. On flood tides, tidal straining is a source of turbulent energy (positive values of  $B_h$ ), and can contribute as much as 50% to the overall turbulent energy.

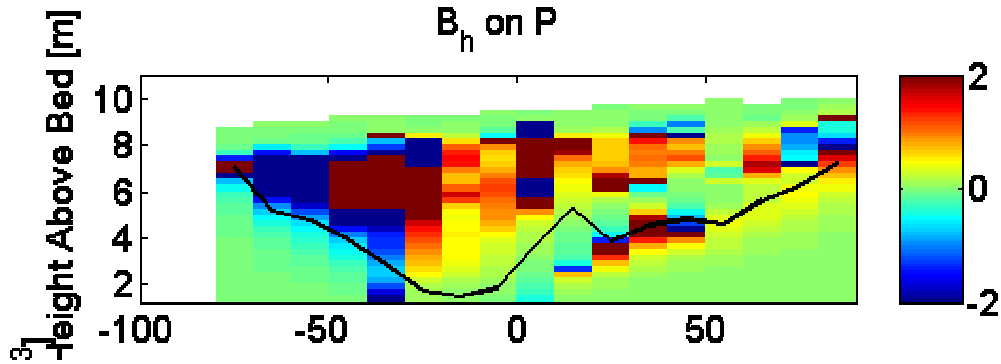
Inclusion of this straining term into the local equilibrium condition for the TKE in an unstratified boundary layer allows us to quantify the resulting ebb-flood asymmetry in the TKE (as is shown in Figures 4a and 4b). Assuming the boundary layer remains unstratified,  $B_v \sim B_h$ , and the local equilibrium condition is that:

$$P + B_h - \varepsilon = 0$$

Where we have assumed that the TKE is steady in the BBL and that advection and diffusion of TKE are negligible. Making the scaling argument that  $\varepsilon \sim q^3/l$ , we can solve for the TKE as:

$$q^2 \approx l^{2/3} P^{2/3} (1 + B_h / P)^{2/3}$$

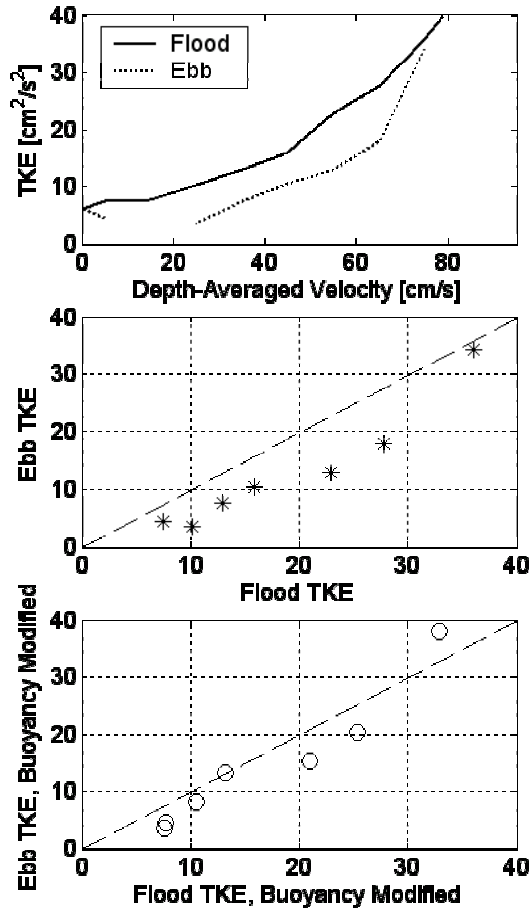
Where the reversing sign of the buoyancy flux between ebb and flood results in an asymmetry in the TKE. To evaluate the magnitude of the strain-induced asymmetry in the TKE, we have normalized the TKE by  $(1 + B_h/P)^{2/3}$ ; the resulting comparison between ebb and flood is shown in figure 4c. The result indicates that the magnitude of the ebb-flood asymmetry is captured in this horizontal buoyancy flux.



**Figure3: Ratio of horizontal buoyancy flux to local shear production based on integration from the bed to height  $z$ . Stabilizing horizontal fluxes are negative, and solid black line shows calculated height of bottom boundary layer.**

## IMPACT/APPLICATIONS

Each of the results described here suggest effective scaling approaches to the prediction of turbulence characteristics in a tidal flow with horizontal density gradients. As such, it suggests that simple parameterizations of turbulent mixing in these shallow flows may be possible, which would be of benefit in the development and application of circulation models for these systems.



**Figure 4: Turbulent Kinetic Energy, comparison of ebb and flood tides. (a) Measured TKE vs. depth-averaged velocity; (b) Direct comparison of ebb and flood TKE levels for equivalent tidal forcing; (c) Same comparison as (b), but after normalizing by  $(1+B_H/P)^{2/3}$ .**

## RELATED PROJECTS

OBSERVATION OF PHYSICAL FLUXES BETWEEN AN ESTUARY AND THE OCEAN (Sea Grant – Stacey and Powell) – Direct observations of fluxes of water, temperature, salinity, and biological scalars during different seasons to determine which physical mechanisms dominate exchanges.

THE ROLE OF FRONTS IN ESTUARINE CIRCULATION AND TRANSPORT (NSF – Stacey) – Examines the effects of short timescale and small scale features on estuarine circulation, transport and mixing.

## PUBLICATIONS

Stacey, M. T. and Ralston D. K. , “The scaling and structure of the estuarine bottom boundary layer,” submitted to *J. Phys. Oceanogr.*, 2003.